Problem to be used on 11/10: 5.4 (Curve Sketching), 6.1 (Absolute Extrema), 6.2 (Applications of Extrema), 6.3 (Further Applications to Economics), 6.4 (Implicit Differentiation)

Group 1

Let $f(x) = e^{-2x}$. For $x > 0$, let $P(x)$ be the perimeter of the rectangle with vertices $(0,0), (x,0), (x, f(x)), (0, f(x))$. Which of the following statements is true?
(a) The function P has an absolute minimum but not an absolute maximum on the interval $(0, \infty)$.
(b) The function P has an absolute maximum but not an absolute minimum on the interval $(0, \infty)$.
(c) The function P has both an absolute minimum and an absolute maximum on the interval $(0, \infty)$.
(d) The function P has neither an absolute maximum nor an absolute minimum on the interval $(0, \infty)$, but the graph of the function P does have an inflection point with positive x-coordinate.
(e) The function P has neither an absolute maximum nor an absolute minimum on the interval $(0, \infty)$, but the graph of the function P does have an inflection point with positive x-coordinate.

Group 2

Application to Business
A local group of scouts has been collecting aluminum cans for recycling. The group has already collected 12,000 lb of cans, for which they could currently receive \$7.50 per hundred pounds. The group can continue to collect cans at the rate of 400 lb per day. However, a glut in the aluminum market has caused the recycling company to announce that it will lower its price, starting immediately, by \$0.15 per hundred pounds per day. The scouts can make only one trip to the recycling centre. Find the best time for the trip. What total income will be received?

Group 3

Let $e^{u^2-v} - v = 1$. Find each derivative.
(a) du/dv
(b) dv/du
(c) What do you notice about the relationship between du/dv and dv/du?

Group 4

Applications to Life Sciences
The equation

$$f(x)g(N) - m - s(x) = 0$$

describes the growth rate of phytoplankton at equilibrium, where x is the phytoplankton cell size, f is the maximum growth rate, N is the nutrient concentration, g represents the nutrient limitation experienced by phytoplankton, m is the mortality rate (a constant), and s is the loss due to sinking. Addressing the question of how phytoplankton evolution affects nutrient concentration requires finding the rate of change of N with respect to x. Using implicit differentiation, show that

$$\frac{dN}{dx} = \frac{s'(x) - f'(x)g(N)}{f(x)g'(N)}$$

Group 5

Sketch the graph of a single function that has all of the properties listed:
(a) Continuous everywhere except at $x = -4$, where there is a vertical asymptote
(b) A y-intercept at $y = -2$
(c) x-intercepts at $x = -3, 1$, and 4
(d) $f'(x) < 0$ on $(-\infty, -5), (-4, -1), (2, \infty)$
(e) $f'(x) > 0$ on $(-5, -4), (-1, 2)$
(f) $f''(x) > 0$ on $(-\infty, -4), (-4, -3)$
(g) $f''(x) < 0$ on $(-3, -1), (-1, \infty)$
(h) Differentiable everywhere except at $x = -4$ and $x = -1$

1 Extra Problems

1. A geometric interpretation of elasticity is as follows. Consider the tangent line to the demand curve $q = f(p)$ at the point $P_0 = (p_0, q_0)$. Let the point where the tangent line intersects the p-axis be called A, and the point where it intersects the q-axis be called B. Let P_0A and P_0B be the distances from P_0 to A and to B, respectively. Calculate

the ratio P_0B/P_0A in terms of p_0, q_0, and $f(p_0)$, and show that this ratio equals the elasticity (assuming $p_0, q_0 \geq 0, f(p_0) \leq 0$, also recall elasticity$= -\frac{p_0}{q_0} f'(p_0)$).

2. Every year, Erin D'Aquanni sells 30,000 cases of her Famous Spaghetti Sauce. It costs her \$1 per year in electricity to store a case, plus she must pay annual warehouse fees of \$2 per case for the maximum number of cases she will store. If it costs her \$750 to set up a production run, plus \$8 per case to manufacture a single case, how many production runs should she have each year to minimise her total costs? (Hint: exercise 15, 17 p.348. If you are interested, solve those two exercises.)